

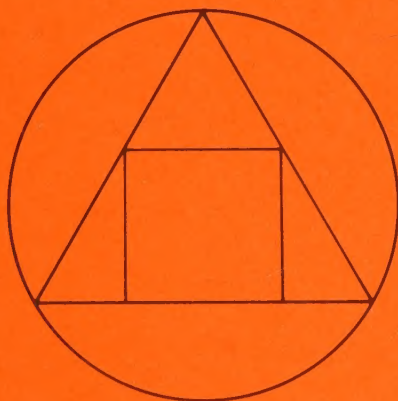
Doctor's thesis

MACHINE ELEMENTS

DIVISION

Lund Technical University

Lund, Sweden



External Involute Gear Stresses

Erik Wennerström

Lund 1973

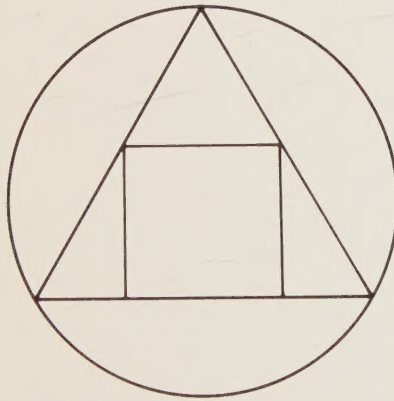
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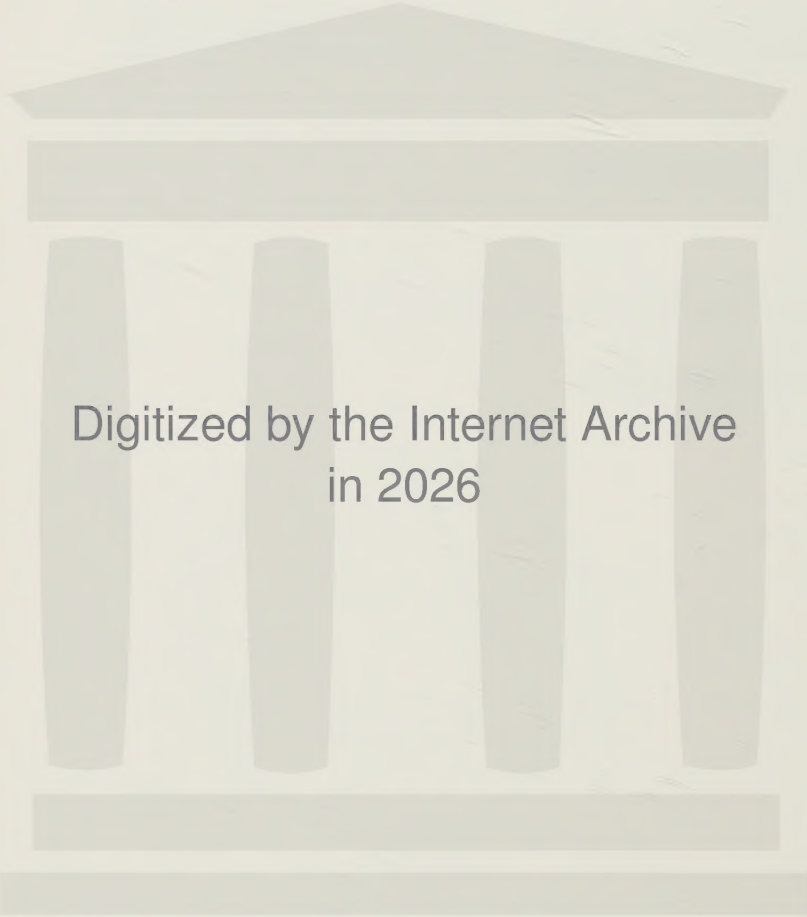
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The following papers are included in this dissertation:

1. Wennerström, Erik: Stresses in External Involute Spur Gears. Acta Polytechnica Scandinavica. Mech. Eng. Series No. 53, Stockholm 1970.
2. Wennerström, Erik: Tension of a Strip with Two Semi-circular Notches. Trans. Machine Elements Division, Lund Technical University. Lund 1973.
3. Wennerström, Erik: On the Optimum Design of External Involute Gears. Trans. Machine Elements Division, Lund Technical University. Lund 1973.

Preface

This booklet is a summary of three papers on gear stresses and stress calculations by the author. These papers are the results of six years' theoretical and experimental gearing research, 1964-1969. The research work was carried out at the Machine Elements Division, Lund Technical University, Lund, Sweden. Professor Leif Floberg, Head of the Division, was research leader. The Swedish Board for Technical Development supported the work.

This is a work on stresses in external involute gears, including a numerical method that solves the conditions of equilibrium and continuity. In order to get a base for an optimum design, new non-dimensional parameters for the calculation of gear bending stresses and gear contact pressures are introduced.

The gear load is constant along the total line of contact and is derived from the transmitted power and the angular velocity, which is of such a magnitude that no dynamic loads are imposed. Gear surface fatigue is supposed to be a problem of contact pressure with no respect to sliding conditions.

Bending stresses are given for a number of spur gear cases and contact pressures for a large number of spur and helical gear cases. A conclusion from all results is that large pressure angles are favourable in heavily loaded gears. The tests made are in very good agreement with the theoretical results.

I am greatly indebted to Professor Leif Floberg for his invaluable ideas, advice, and encouragement. A support without which this work could never have been realized.

I want to express my gratitude to the Swedish Board for Technical Development for their sponsorship.

Erik Wennerström

Stresses in External Involute Spur Gears

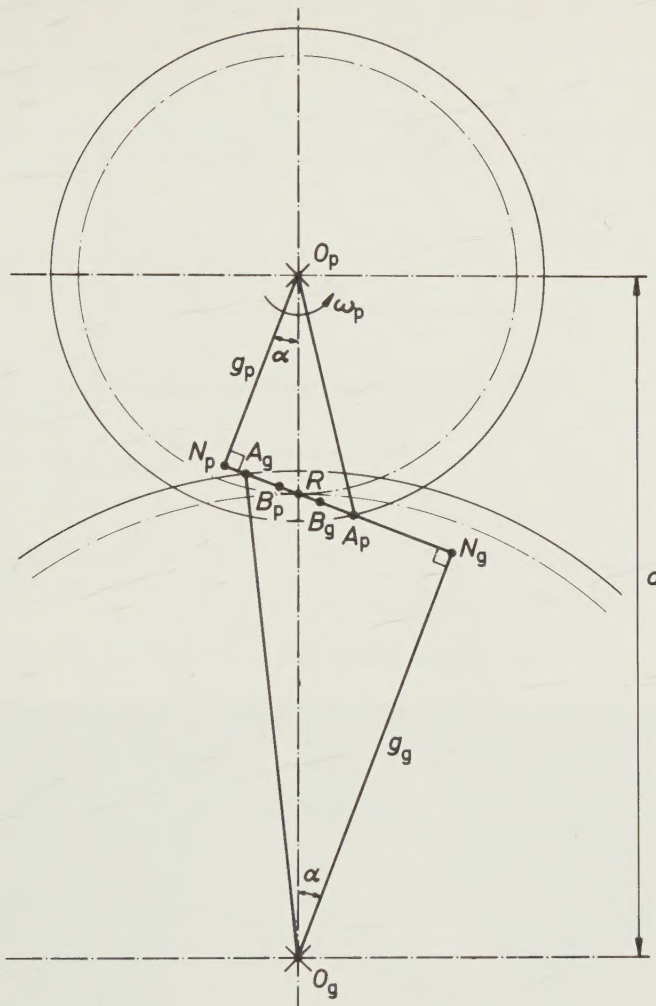


Fig. 1. Geometry of the external involute gear

This gear can be designed in a great many ways. To be able to compare different designs we have to conclude the comparison conditions. These are: equal gear ratio, equal pinion torque, equal centre distance, and equal width of gear faces. The gear stress is now expressed in these parameters and a non-dimensional stress, different for bending stress and contact pressure.

One of the gear teeth in fig. 2 is loaded by a force and the stress distribution within this tooth is solved on the basis of the equilibrium and continuity conditions of the elastic gear material.

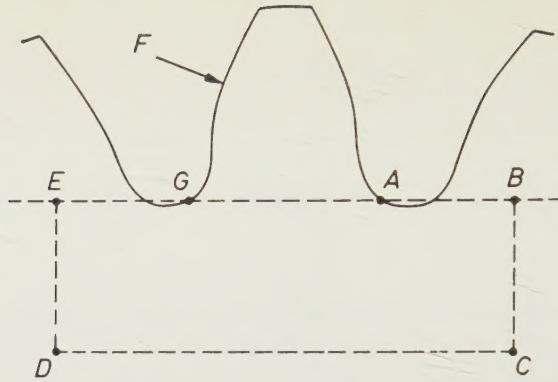


Fig. 2. A gear tooth carrying the load F

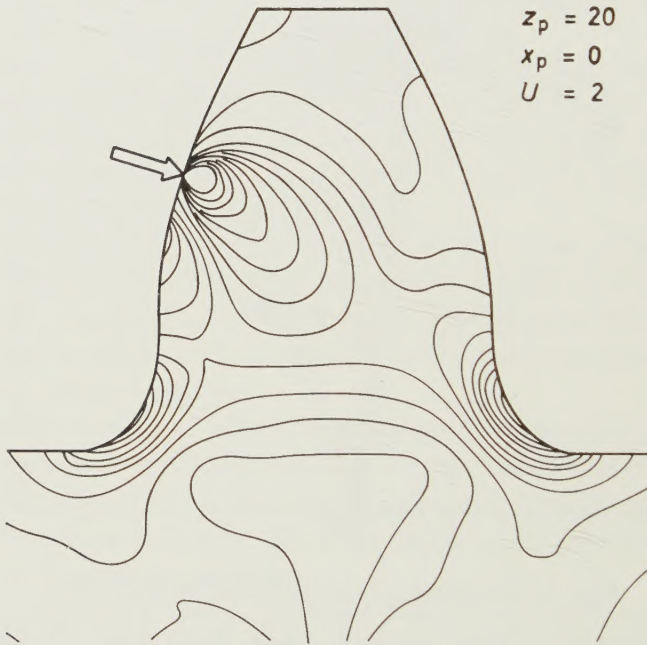


Fig. 3. Theoretical stress curves $\sigma_{01} - \sigma_{02} = 25$ i

The curves of equal difference of the principal stresses in fig. 3 are also the maximum shearing stresses within the tooth and the tangential stresses in the gear surface.

This stress distribution has been tested on a photoelastic model under a predicted load. Fig. 4 represents the theoretical solution and fig. 5 the test result.

The non-dimensional bending stress is $\sigma_0 = \sigma \omega_p a^2 b / P$ and should be as low as possible.

A large number of cases have been calculated and all results are within the range of $1.22 z_p (1 + U)^2 \leq \sigma_0 \leq 1.75 z_p (1 + U)^2$. This means that the bending stress is an approximate linear function of the number of teeth. It also gives an approximate estimation of the bending stress $\sigma = 1.75 z_p (1 + U)^2 P / \omega_p a^2 b$.

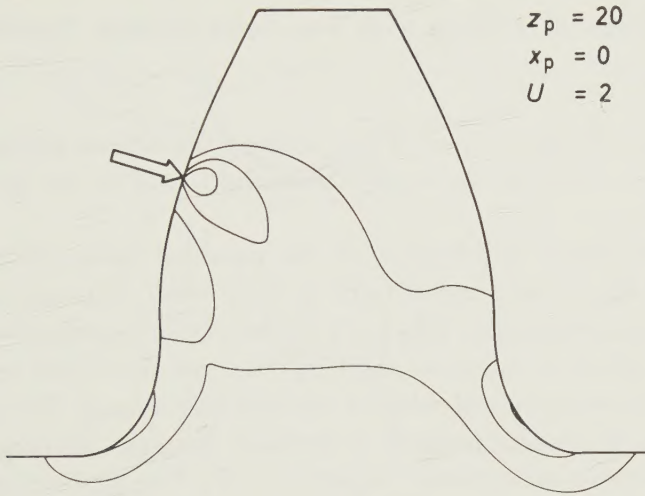


Fig. 4. Theoretical stress curves $\sigma_{01} - \sigma_{02} = 100$ i

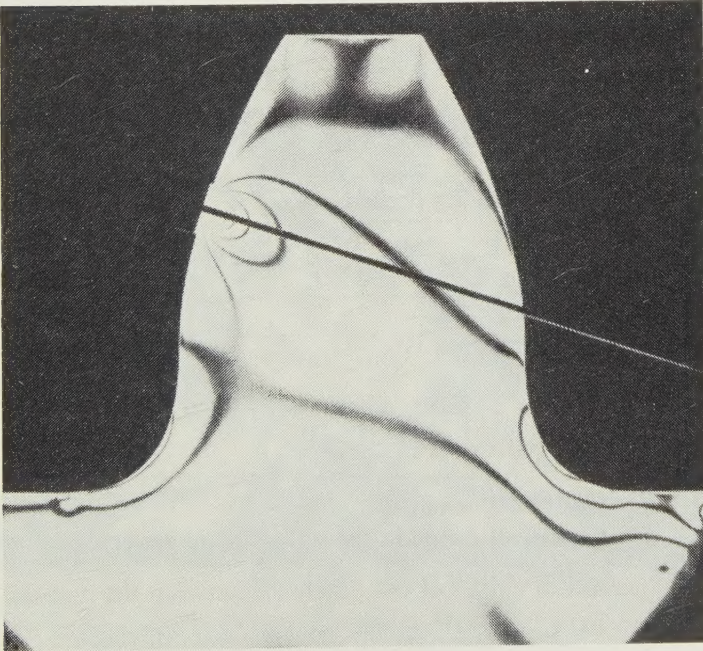


Fig. 5. Photoelastic model loaded by $F=718$ N

Tension of a Strip with Two Semi-circular Notches

The analytical solution of this problem was known earlier. It was studied as a test of the numerical method used in the gear stress calculation.

A theoretical distribution of the principal stress difference is shown in fig. 7, for a radius ratio $r_0 = r/b = 0.5$. This case was also studied photoelastically. Fig. 8 is a record of that experiment.

Five different ratios of radius r_0 was calculated and compared to the correct analytical solution by Isida and Atsumi. The comparison in fig. 6 was also made to theoretical colutions by Ling and by Neuber and to photoelastic experiments by Frocht, Guernsey, and Landsberg.

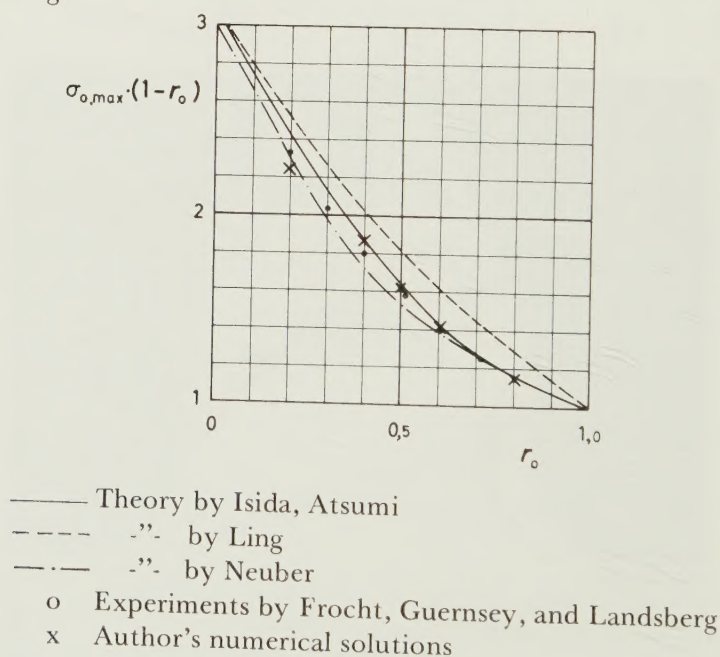


Fig. 6. Comparison of different solutions

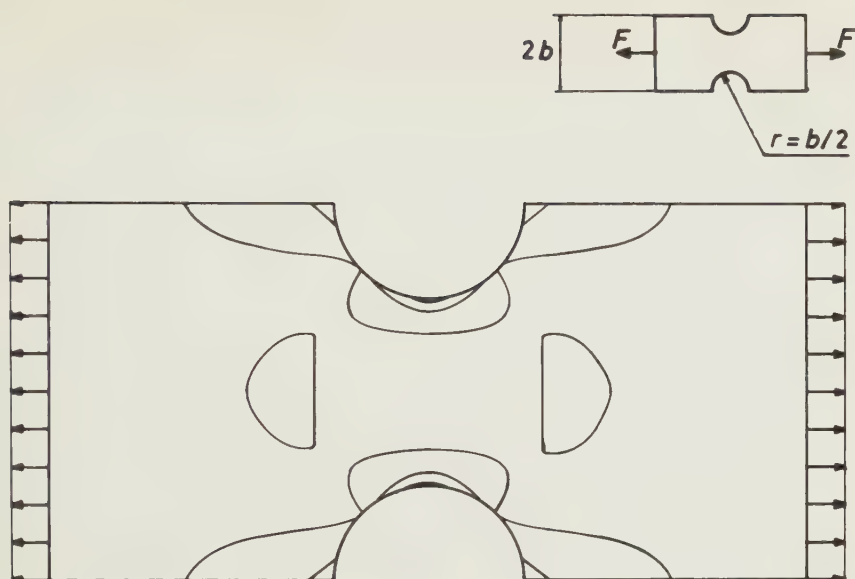


Fig. 7. Theoretical isochromatics $\sigma_{01} - \sigma_{02} = 0.75 i$

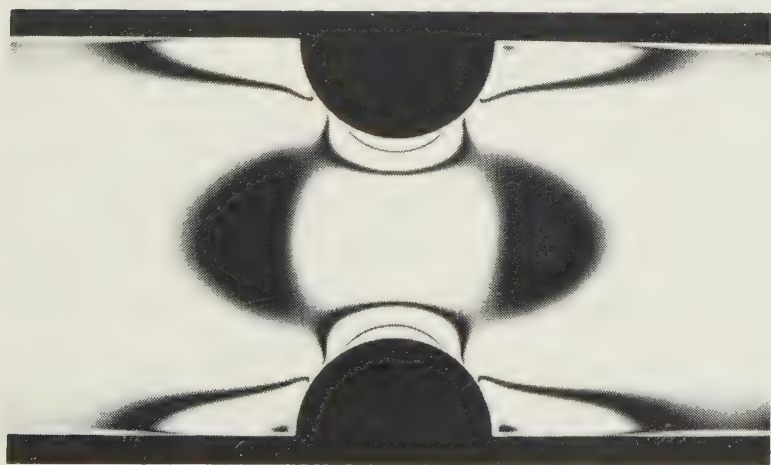


Fig. 8. Photoelastic experiment $F=1334 \text{ N}$, $r_0=0.50$

On the Optimum Design of External Involute Gears

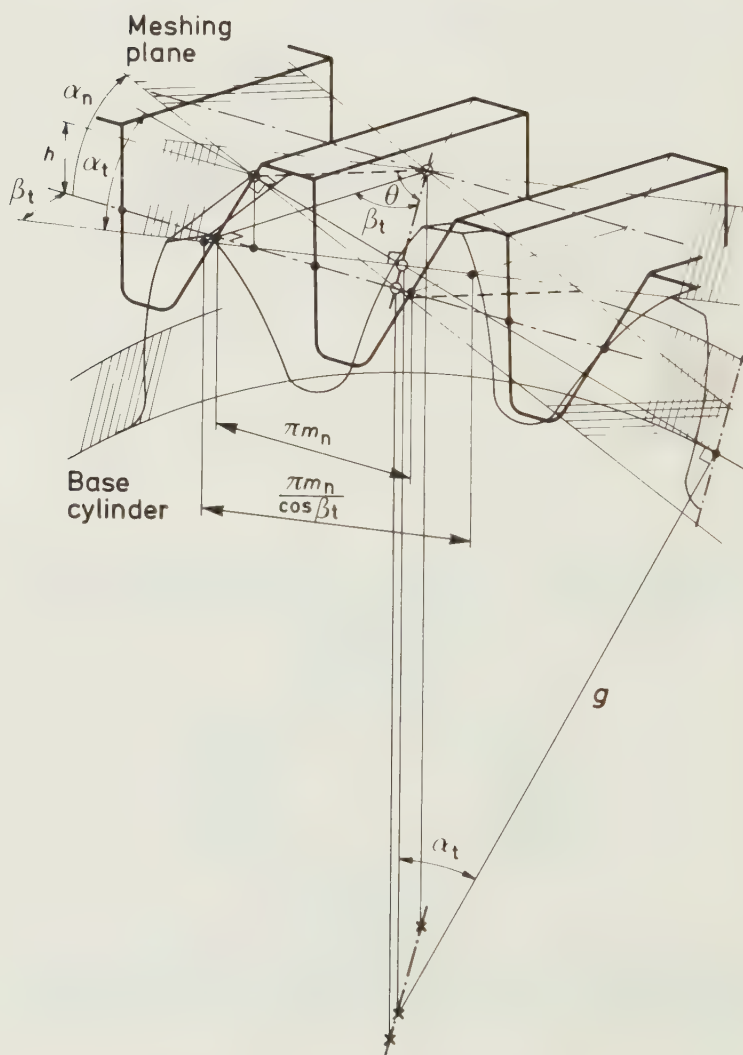


Fig. 9. Helical gear during rack tool generation

The non-dimensional gear contact pressure is $p_{o,max} = p_{max} \sqrt{\omega_p a^2 b (k_p + k_g) / P}$. This contact pressure is calculated in a large number of cases. For the design of a gear with regard to surface fatigue, $p_{o,max}$ should be as low as possible. Fig. 10 is a set of spur gear results. It is seen that the contact pressure decreases rapidly with an increasing number of teeth to about $z_p = 20$ pinion teeth and then remains fairly constant. There is in fact an optimum point in the vicinity of $z_p = 20$ provided that the pinion is sufficiently addendum modified. These optimum points are shown in fig. 11, which gives a value for the lowest obtainable gear contact pressure. The gear bending stresses are almost linear to the number of teeth z_p . Hence there is always a balancing point between optimum bending stress and optimum contact pressure depending on the resistance of the gear material to breakage versus surface failure. Cast iron and plastic gears are sensitive to gear breakage and thus have to be designed with a low number of pinion teeth.

Helical gears have no such optimum number of pinion teeth. Instead there is an optimum helical angle. With the standard tool angle $\alpha_n = 20^\circ$ the optimum helical angle $\beta = 50^\circ$ is not usable as it causes bearing difficulties by high axial loads. An increase of the tool pressure angle decreases the optimum helical angle. Also gear bending stresses decrease. The author suggests a wider use of higher tool pressure angles for heavily loaded gears.

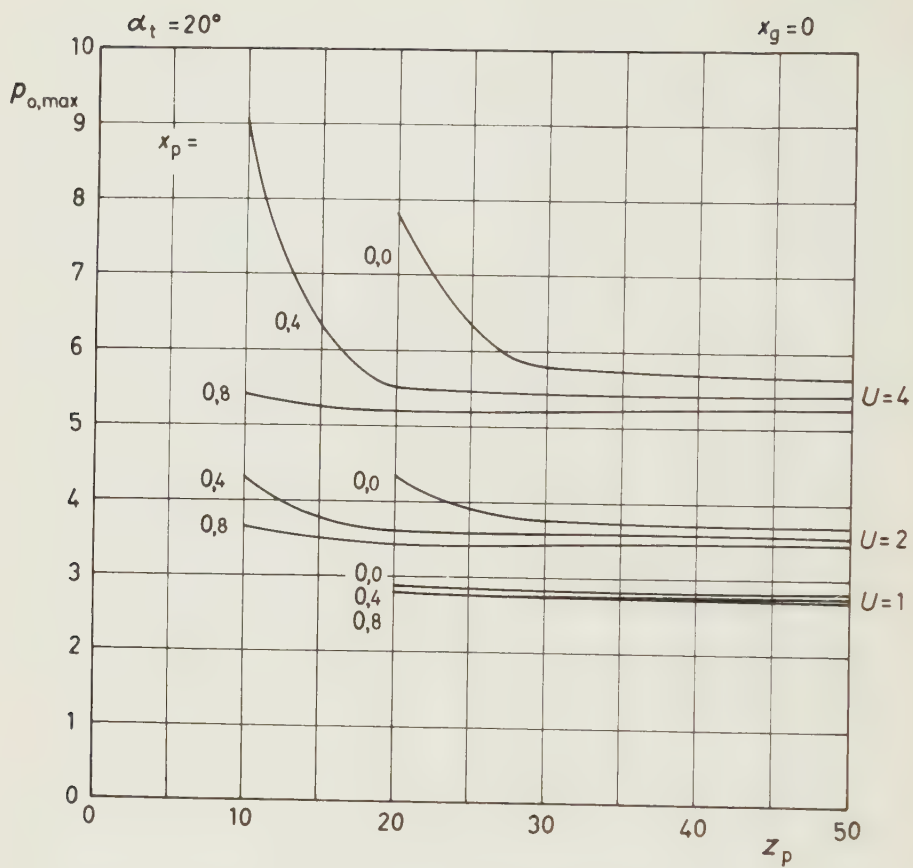


Fig. 10. Non-dimensional spur gear contact pressures

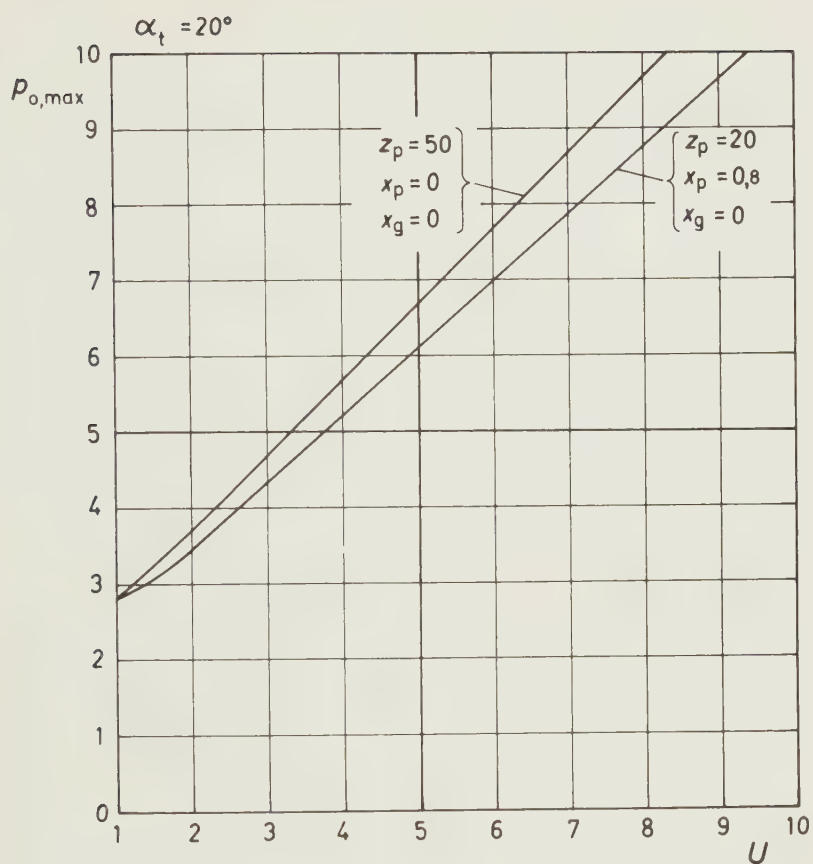


Fig. 11. Optimum spur gear contact pressures

